

Topic Proposal:

Complex Hyperbolic Geometry

RTG Log Cabin Seminar Winter 2023

Complex hyperbolic geometry is an analogue of (real) hyperbolic geometry and delves into different areas of mathematics. On the one hand, one can define complex hyperbolic space in a down to earth way as a level set of a Hermitian form on \mathbb{C}^3 . It can then be studied as a (real) Riemannian manifold (of variable curvature), but also as a complex manifold (with constant holomorphic curvature). More than that, it carries a symplectic structure and can even be seen as a Kähler manifold. The boundary of complex hyperbolic space can be identified with the Heisenberg group, endowing the space with even more structure.

On the other hand, this space is an example of a symmetric space, with isometry group $\mathrm{PU}(n, 1)$. A complex hyperbolic manifold can thus be thought of a quotient of $\mathrm{PU}(n, 1)$ by one of its discrete subgroups. In this language, one can ask about properties of discrete subgroups of Lie groups, and recent papers explore the **rigidity** of such subgroups (how much they can deformed), and **how they can be constructed**.

The goal of the seminar is to understand complex hyperbolic space, and use this model to understand some aspects of complex geometry, symplectic and contact geometry, symmetric spaces and Lie groups, and possibly explore new results in this area. We would be following **John Parker's notes** and chapter 10 of the **book by Bridson-Haefliger's** for the basics, Goldman's book on *Complex Hyperbolic Geometry* for more in depth topics, and the above papers for the new results on rigidity and construction of complex hyperbolic lattices.