Boundaries

RTG Log Cabin Seminar Fall 2023

Topic Proposal

The concept of geometric boundaries captures the asymptotic behavior of various geometric objects such as groups, spaces and manifolds. The set of boundary points offers ways to compactify our object of interest and above all, it often carries surprising geometry itself. However, the notion of a boundary is far from being canonical even for the same type of objects – there may be many inequivalent boundaries arising from different contexts.

The goal of this seminar is to introduce the different notions of boundaries in broad brushstrokes, with a focus on examples and applications. We will try to understand the motivation behind these definitions and discover how they evolve as we relax the rigidity of our objects. Furthermore, we will find out how boundaries can reveal more information about the objects we wish to study and see what keeps hidden.

This topic lies in the intersection of many aspects of geometry and may yield something useful for everyone in Heidelberg and Karlsruhe. We aim to divide the topics in a way that strikes a balance between the interests of both groups. Please send us your questions and comments to the contacts [1] [2] below. The following is a list of topics we would like to cover, which is of course open to further suggestions.

Basic Topics: (2 talks each)

- Visual boundary for Hadamard manifolds

- Gromov boundary for hyperbolic groups

- Furstenberg boundary and flag manifolds for symmetric spaces

Advanced/Optional Topics: (1-2 talks each)

- Morse boundary for proper geodesic metric spaces

- Shilov boundary for Hermitian symmetric spaces

- Tits boundary and buildings

- Poisson boundary and Martin boundary for asymptotic behavior of random walks

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[1]Contact: Enya Hsiao, ehsiao@mathi.uni-heidelberg.de
[2]Contact: Julia Heller, julia.heller@kit.edu

For references, it is instructive to do a quick search on wikipedia. We offer here two giant books: [BH99] [Ebe97], and a survey paper: [KB02].

References

[BH99]   Martin R. Bridson and André Haefliger. *Metric spaces of non-positive curvature*. Springer, 1999.

[Ebe97]   Patrick Eberlein. *Geometry of nonpositively curved manifolds*. University of Chicago Press, 1997.

[KB02]   Ilya Kapovich and Nadia Benakli. "Boundaries of hyperbolic groups". In: *Contemporary Mathematics* (2002), pp. 39–93. DOI: `10.1090/conm/296/05068`.