Characteristic Classes

RTG Seminar WS 2018

The seminar intends to introduce characteristic classes as a important tool of modern mathematics and show some applications of this theory.
As references I would suggest Milnor and Stasheff 1974, Cohen 2002 and Hatcher 2003.

Schedule

	We. 11.04.	Thu. 12.04
09:30-10:30	Talk 1 **Menelaos**	Talk 5 **Hartwig**
11:00-12:00	Talk 2 **Mareike**	Talk 6 **Karla**
12:00-14:00	Lunch Break	Lunch Break
14:00-15:00	Talk 3 **Anna-Maria**	Talk 7/8 **Urs**
15:30-16:30	Talk 4 **Pascal**	Talk 9/10 **Johannes**

All talks will take place at **seminar room 4** on the 3rd floor of the Mathematikon.

List of talks

The talks signed with * should definitely be covered. The others concern additional topics or interesting applications and are more or less independent.

1* **Vector Bundles and Principal Bundles**
Fiber bundles, real and complex vector bundles, principal bundles, associated bundles, Examples. Milnor and Stasheff 1974 § 2,13, Hatcher 2003 § 1.1, Cohen 2002 §1.1

2* **Operations on Bundles**
Pullback, direct sum, Tensor product, (quotient bundle, Picard variety, ortho. complement, Functoriality, Kernel, Image of homo).. Milnor and Stasheff 1974 § 3, Hatcher 2003 § 1.1

3* **Stiefel-Whitney classes and Chern classes**
Axioms, easy consequences, existence of sections vanishing of w_i, total characteristic classes, (Whitney duality, Thom-Pontrjagin theorem, Cobordism Theorem).Hatcher 2003 § 3, Milnor and Stasheff 1974 § 4,16

4* **Proof of existence and uniqueness of Stiefel-Whitney and Chern classes**
Hatcher 2003 § 3

5* **Euler class and Pontrjagin class**
oriented vector bundles, Fundamental class, Euler class, Euler class vs. Stiefel-Whitney class, Euler class of tangent bundle, Pontrjagin classes, Pontrjagin classes vs. Stiefel-Whitney class. Milnor and Stasheff 1974 § 9,11,15, Hatcher 2003 3.2, Cohen 2002 §3.5

6 **Characteristic classes as Obstructions**
Hatcher 2003 §3 Intro., Milnor and Stasheff 1974 §12

7 **Classification of bundles 1**
Homotopy invariance of pullback, Def. universal bundles, (Class. of vb. over spheres), aspherical ⇒ unviersal bundle, infinite Grassmanians. Cohen 2002 §2.1, 2.2

8 **Classification of bundles 2**
Eilenberg MacLane spaces, Classification of line bundles, Existence of universal spaces, Join construction, (applications), characteristic classes as generators of Cohomo. of classifying spaces. Cohen 2002 § 2.2,2.4

9 **Connections on vector bundles and Chern-Weil theory 1**
connection on vb, exterior derivative, pullback, curvature, inv. polynomials, Def. of characteristic classes in terms of curvature of complex vb with connection. Milnor and Stasheff 1974 App. C,Cohen 2002 §3.6

10 **Chern-Weil theory 2 and generalized Gauß-Bonnet**
Identifying Chern classes/ Pontrjagin classes/ Euler class in terms of curvature, Generalized Gauß-Bonnet, (Outlook on Index theory). Milnor and Stasheff 1974 App. C, Cohen 2002 §3.6

References

Cohen, Ralph L. (2002). *The Topology of Fiber Bundles*. URL: http://math.stanford.edu/~ralph/fiber.pdf.

Hatcher, Allen (2003). *Vector Bundles and K-Theory*. URL: https://www.math.cornell.edu/~hatcher/VBKT/VB.pdf.

Milnor, John W. and James D. Stasheff (1974). *Characteristic classes.* English. Annals of Mathematics Studies. No.76. Princeton, N.J.: Princeton University Press and University of Tokyo Press. VII, 331 p. $ 10.00 (1974).