Abstract: One natural way to explore the structure of an infinite discrete group is to study its proper quotient groups. This problem often splits into two subproblems that happen to require fundamentally different methods: the study of finite quotients versus that of infinite quotients. The goal of this talk is to illustrate this paradigm by means of two classes of groups: multiplicative groups of division algebras over number fields, and Gromov hyperbolic groups.